Formal Mathematical Proofs for the RMAχτ Coherence Framework

Three Papers Establishing the Quantum Information-Theoretic Foundation

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Bundle Contents

This document contains three formally interconnected papers that establish the quantum information-theoretic foundation for the RMAχτ coherence framework:

1. **Born Rule Uniqueness** — Derives P = |psi|^2 as the unique projection survivor, not an independent axiom
2. **Q <-> M Equivalence** — Establishes the quantum potential Q and suppression coefficient M® as complementary partial traces of the same interaction Hamiltonian
3. **CPTP Projection Map Formalization** — Embeds the entire projection process within standard quantum channel theory, with the fossilization-recycling decomposition as an additional physical hypothesis

The three papers form a triad: Born Rule establishes what survives projection, Q<->M establishes how quantum and classical descriptions connect, and CPTP provides the formal scaffold ensuring compatibility with standard quantum mechanics.

PAPER 1

Born Rule Uniqueness: $|\psi|^2$ as the Unique Projection Survivor

Jason Smith, Charles Borabon

1. The Problem

The Born rule — $P(i) = |\langle i|\psi\rangle|^2$ — is the last independent axiom of quantum mechanics. It asserts that the probability of measuring outcome i is the squared modulus of the corresponding amplitude. Every standard formulation treats this as a postulate bolted onto the Schrödinger equation.

Previous derivation attempts:

- **Gleason (1957):** Proved Born rule is the unique probability measure on Hilbert space projectors (dim ≥ 3). Assumes probability measures exist and satisfy additivity — equivalent to assuming conservation structure.
- **Zurek (envariance):** Uses environment-assisted invariance in the decoherence context. Requires "equal probability for equal coefficients" as input — a symmetry assumption equivalent to the result.
- **Deutsch-Wallace (many-worlds decision theory):** Derives Born rule from rational betting preferences. Requires many-worlds ontology and rational agent axioms.

Each correctly identifies part of the structure but smuggles in an assumption equivalent to what it tries to prove.

2. Framework Position

The RMAχτ framework claims: when Layer 0 (coherence manifold) projects to Layer 1 (classical reality), phase information S is exported to the projection medium. What survives is ρ_c — identified with $|\psi|^2$.

The claim in the Full Circuit paper was: "The Born rule is what happens when you project out the phase." This paper proves that claim by establishing **uniqueness** — $|\psi|^2$ is not merely a convenient choice but the ONLY quantity consistent with the projection process, within a precisely specified class of admissible candidates.

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3. The Four Constraints

The projection process (Layer 0 → Layer 1) imposes four physical constraints on whatever quantity survives:

Constraint 1 — Phase Independence

Phase S is exported to the projection medium. The surviving quantity cannot depend on S. It must be constructible from $|\psi|$ alone.

Formal statement: The surviving quantity ρ must satisfy $\rho(\psi) = \rho(\psi \cdot e^{i\theta})$ for all θ. This restricts ρ to functions of $|\psi|$ only.

Constraint 2 — Non-Negativity and State Dependence

The surviving quantity represents coherence density. $\rho \geq 0$ always, and ρ must depend on the quantum state — it cannot be a constant independent of ψ.

Formal statement: $\rho = f(|\psi|)$ with $f: \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, f not identically constant.

Constraint 3 — Local Conservation

The surviving quantity must satisfy a **local** continuity equation — not merely global conservation. This is the standard requirement for probability in quantum mechanics: probability is locally conserved, meaning it cannot disappear from one region and reappear in a distant region without flowing through the intervening space.

Formal statement: There exists a current density **j**_ρ such that:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j}_\rho = 0 \tag{C3}$$

everywhere, for all ψ and V.

We require local conservation (C3), NOT merely:

$$\frac{d}{dt}\int \rho \, d^3x = 0$$

The integrated form would permit source-sink pairs (ρ appearing in one region and vanishing in another) as long as the total is conserved. Local conservation forbids this.

Constraint 4 — Derivability from Schrödinger Dynamics (Admissible Class)

The continuity equation for ρ, including its current **j**_ρ, must follow from the Schrödinger equation itself — no external input.

Admissible density class: ρ = f(|ψ|) for some smooth function f: $\mathbb{R}$≥0 → $\mathbb{R}$≥0.

Admissible current class: **j**_ρ must be derivable from Schrödinger dynamics by algebraic manipulation of iℏ ∂ψ/∂t = Hψ and its conjugate. Because the Schrödinger equation is linear in ψ, its conjugate is linear in ψ*, and the Hamiltonian H = −ℏ²/(2m)∇² + V introduces at most second spatial derivatives, any current obtained by this procedure is necessarily:

- **Bilinear** in (ψ, ψ*, ∇ψ, ∇ψ*) — because the generating equations are linear in ψ and ψ* separately, and any time derivative of f(|ψ|) involves products of ψ-terms and ψ*-terms.
- **Local** — depending only on field values and their derivatives at each point, because the Hamiltonian is a local differential operator.
- **First-order in spatial derivatives** — because expressing the result as ∇·**j**_ρ requires one integration of the ∇² terms from the kinetic energy operator.

This is the same structural class as the standard probability current (equation 4.2). It is not an independent restriction — it is the class that Schrödinger dynamics generates. Exotic alternatives (non-local functionals, higher-order derivatives, fractional operators) would require dynamical input beyond the Schrödinger equation and are therefore excluded by the derivability requirement, not by fiat.

Formal statement: The continuity equation (C3) with admissible ρ and j_ρ is derivable from iℏ ∂ψ/∂t = Hψ and its conjugate, with H Hermitian, for general potentials V(x) and general wavefunctions ψ(x,t).

4. The Derivation

4.1 Candidate Space

Given Constraints 1 and 2, the surviving quantity must be a non-negative, state-dependent function of |ψ|. The natural candidates are power laws:

$$\rho_p = |\psi|^p \quad \text{for } p > 0 \tag{4.1}$$

More general functions f(|ψ|) can be considered, but the continuity equation analysis depends on how ρ scales with |ψ|. Any smooth non-negative f with f(0) = 0 has leading behavior ∝ |ψ|^p near zeros of ψ, and the source-free condition applies to this leading behavior. The power-law family is therefore the natural and sufficient class to analyze. We state the theorem for this class; the generalization is discussed in §4.7.

4.2 The Standard Continuity Equation (p = 2)

Start from the Schrödinger equation and its conjugate:

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi, \qquad -i\hbar\frac{\partial\psi^*}{\partial t} = H\psi^*$$

where $H = -\hbar^2/(2m)\nabla^2 + V(x)$ with V real.

Multiply the first equation by ψ*, the second by ψ, subtract:

$$i\hbar\frac{\partial|\psi|^2}{\partial t} = -\frac{\hbar^2}{2m}\left(\psi^*\nabla^2\psi - \psi\nabla^2\psi^*\right) = -\frac{\hbar^2}{2m}\nabla\cdot(\psi^*\nabla\psi - \psi\nabla\psi^*)$$

(The V terms cancel: $V|\psi|^2 - V|\psi|^2 = 0$.)

Define the probability current:

$$\mathbf{j} = \frac{\hbar}{2mi}(\psi^*\nabla\psi - \psi\nabla\psi^*) \tag{4.2}$$

Result:

$$\boxed{\frac{\partial|\psi|^2}{\partial t} + \nabla\cdot\mathbf{j} = 0} \tag{4.3}$$

This is a source-free local continuity equation. **p = 2 satisfies all four constraints.** The current **j** is bilinear in (ψ, ψ*, ∇ψ, ∇ψ*) — within the admissible class.

4.3 General Power Test (p ≠ 2)

Now test whether $|\psi|^p$ for arbitrary $p > 0$ satisfies a source-free local continuity equation derivable from Schrödinger dynamics.

Compute the time derivative:

$$\frac{\partial|\psi|^p}{\partial t} = \frac{p}{2}|\psi|^{p-2}\cdot\frac{\partial|\psi|^2}{\partial t} \tag{4.4}$$

Substitute the known result for $|\psi|^2$:

$$\frac{\partial|\psi|^p}{\partial t} = -\frac{p}{2}|\psi|^{p-2}\nabla\cdot\mathbf{j} \tag{4.5}$$

Expand using the product rule:

$$= -\frac{p}{2}\left[\nabla\cdot\left(|\psi|^{p-2}\mathbf{j}\right) - \mathbf{j}\cdot\nabla\left(|\psi|^{p-2}\right)\right] \tag{4.6}$$

$$= -\frac{p}{2}\nabla\cdot\left(|\psi|^{p-2}\mathbf{j}\right) + \frac{p}{2}\mathbf{j}\cdot\nabla\left(|\psi|^{p-2}\right) \tag{4.7}$$

For this to be a **source-free local continuity equation** (pure divergence, no leftover source term), the second term must vanish:

$$\frac{p}{2}\mathbf{j} \cdot \nabla \left(|\psi|^{p-2}\right) = 0 \quad \text{for all } \psi, V, t \tag{4.8}$$

4.4 Analysis of the Source Term

Case p = 2: Then |ψ|^{p−2} = $|\psi|^0$ = 1 (constant). ∇(1) = 0. Source term vanishes identically. ✓

Case p = 0: Then ρ_0 = $|\psi|^0$ = 1 for all ψ. This density is constant, independent of the quantum state. It fails Constraint 2 (state dependence): a density that carries no information about ψ cannot serve as a probability measure. ✗

Case p ≠ 0, 2: Then |ψ|^{p−2} is generically non-constant (it varies wherever |ψ| varies). The gradient ∇(|ψ|^{p−2}) ≠ 0 in general.

The source term vanishes only if **j** ⊥ ∇|ψ|^{p−2} everywhere at all times.

4.5 Proof That p ≠ 2 Fails Generically

Write ψ = |ψ| · e^{iS/ℏ} (Madelung form). Then:

$$\mathbf{j} = \frac{|\psi|^2}{m}\nabla S \tag{4.9}$$

$$\nabla \left(|\psi|^{p-2}\right) = \frac{p-2}{2}|\psi|^{p-4}\nabla \left(|\psi|^2\right) \tag{4.10}$$

The source term becomes:

$$\frac{p(p-2)}{4m}|\psi|^{p-2} \left(\nabla S \cdot \nabla|\psi|^2\right) \tag{4.11}$$

This vanishes if and only if:

(a) p = 0 (excluded: state-independent), or

(b) p = 2 (our target), or

© $\nabla S \cdot \nabla|\psi|^2 = 0$ everywhere for all time — i.e., surfaces of constant phase are everywhere perpendicular to surfaces of constant amplitude.

Condition © is **not generally satisfied**. Consider a Gaussian wavepacket:

$$\psi(x,t) = A \cdot \exp\left(-\frac{(x-x_0)^2}{4\sigma^2} + ik_0 x\right)$$

This has $\nabla S \propto k_0\, \hat{x}$ and $\nabla|\psi|^2 \propto (x-x_0)\, \hat{x}$, which are parallel for $x \neq x_0$. Condition © fails immediately.

More formally: the set of potentials $V(x)$ and initial conditions $\psi(x,0)$ for which $\nabla S \cdot \nabla|\psi|^2 = 0$ for all x and all t has **measure zero** in the space of quantum systems. It requires the phase and amplitude to be functionally independent in a very specific geometric sense that generic dynamics violate.

4.6 Lemma: No Alternative Current Rescues p ≠ 2

A critic might object: "Perhaps there exists some other current **J**_p ≠ (p/2)|ψ|^{p−2} **j** that makes ∂_t|ψ|^p + ∇·**J**_p = 0 identically."

Lemma. Within the admissible current class (local, bilinear in ψ, ψ^*, $\nabla\psi$, $\nabla\psi^*$), the only density $\rho = f(|\psi|)$ admitting a source-free local continuity equation for all solutions of the Schrödinger equation is $\rho = c|\psi|^2$ for constant $c > 0$.

Proof sketch. Any local bilinear current has the form:

$$\mathbf{J} = \alpha(|\psi|) \cdot \mathrm{Im}(\psi^* \nabla \psi) + \beta(|\psi|) \cdot \nabla|\psi|^2 \qquad (4.12)$$

The first term is proportional to the probability current **j**; the second is a "diffusion-like" term proportional to the amplitude gradient.

Computing ∇·**J** and requiring ∂_t f(|ψ|) + ∇·**J** = 0 for all ψ satisfying Schrödinger:

The ∂_t f term contains both the ($\nabla S \cdot \nabla|\psi|^2$) coupling and a $\nabla^2|\psi|^2$ term. The divergence of **J** generates terms involving $\nabla^2 S$ (from the first component) and $\nabla^2|\psi|^2$ (from the second).

Matching the $\nabla^2 S$ terms requires $\alpha(|\psi|) = f'(|\psi|)\cdot|\psi|/(2m)$. Matching the $\nabla^2|\psi|^2$ terms determines β. But the cross-term ($\nabla S \cdot \nabla|\psi|^2$) survives unless $f(|\psi|) = c|\psi|^2$, because this is the only function for which $f'(|\psi|)\cdot|\psi|$ is proportional to $|\psi|^2$ (making the cross-term absorbable into the divergence).

Specifically: $f'(|\psi|)\cdot|\psi| = c_1\cdot|\psi|^2$ requires $f'(x) = c_1\cdot x$, which integrates to $f(x) = (c_1/2)x^2 + c_0$. With $f(0) = 0$ (normalizability), $c_0 = 0$. Therefore $f(|\psi|) = (c_1/2)|\psi|^2 = c|\psi|^2$.

No alternative current within the admissible class rescues any other power or function. ■

4.7 Note on General f(|ψ|)

The power-law analysis (§4.3–4.5) covers the family |ψ|^p. The lemma (§4.6) extends to general smooth f(|ψ|) within the admissible current class. Together, they establish that $|\psi|^2$ is unique not just among power laws but among all smooth, phase-independent, state-dependent densities admitting a source-free local continuity equation with an admissible current.

4.8 Result

Theorem (Born Rule Uniqueness).

Given:

1. *Unitary Layer 0 dynamics governed by the Schrödinger equation with Hermitian Hamiltonian,*
2. *Phase independence: the surviving quantity depends only on |ψ|,*
3. *Non-negativity and state dependence: $\rho = f(|\psi|) \geq 0$, not identically constant,*
4. *Local conservation: ρ satisfies a source-free local continuity equation $\partial\rho/\partial t + \nabla\cdot\mathbf{j}\_\rho = 0$ for all ψ and V,*
5. *Admissible current: **j**_ρ is local and bilinear in (ψ, ψ, ∇ψ, ∇ψ*),**

Then: $\rho = c|\psi|^2$ for some constant $c > 0$, uniquely (up to normalization).

What this proves: $|\psi|^2$ is the unique locally conserved, phase-independent survivor under Schrödinger dynamics, within the class of densities f(|ψ|) and bilinear local currents.

What this does not claim: A fully general functional uniqueness theorem over all conceivable density-current pairs without the admissible class restriction. The restriction to bilinear local currents (the same structural class as the standard probability current) is physically motivated — it excludes exotic constructions involving non-local functionals or higher derivatives that have no physical realization in the projection process.

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5. Framework Interpretation

5.1 What This Means

The Born rule is not an independent axiom. It is a **theorem of the projection process**:

- **Input:** Schrödinger dynamics (Layer 0) + phase discarding (projection mechanism) + local conservation + admissible current class
- **Output:** $|\psi|^2$ uniquely selected as the surviving quantity

No symmetry assumptions (contra Zurek). No decision theory (contra Deutsch-Wallace). No abstract measure theory without physical grounding (contra Gleason). The physics of projection, combined with standard locality and bilinearity requirements, determines the Born rule.

5.2 Connection to the Gradient Equation

The gradient equation (Paper-The RMAχτ Gradient Equation):

$$\frac{d\rho_c}{dr} = -M(r) \cdot \rho_c + R(r) - A_{\chi\tau}(r)$$

is the steady-state, reaction-dominated limit of a reaction-diffusion equation (Paper-The The RMAχτ Gradient Equation, §6). That equation requires a **locally conserved field** — something with a continuity equation. The projection process discards phase. The unique locally conserved, phase-independent, non-negative, state-dependent quantity derivable from Schrödinger dynamics with admissible currents is $|\psi|^2$.

Therefore: **ρ_c IS $|\psi|^2$** — not by analogy, not by choice, but by mathematical necessity within the admissible class.

5.3 Architectural Position

```
Layer 0:   ψ evolves via Schrödinger (given)
              ↓
Projection:  Phase S discarded (framework mechanism)
              ↓
UNIQUENESS:  |ψ|² is the ONLY survivor (this paper, proven)
              ↓
Efficiency:  2/3 of S recycles, 1/3 fossilizes (ϒ, from Landauer)
              ↓
Layer 1:     ρ_c = |ψ|² evolves via gradient equation (validated)
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The Born rule is structurally prior to the recycling efficiency. First you establish WHAT survives ($|\psi|^2$, uniquely); then you ask how much of the discarded phase recycles (ϒ) versus fossilizes (1−ϒ).

5.4 Why |ψ|² and Not Something Else

The uniqueness has a simple physical interpretation. The source term for general p is:

$$\frac{p(p-2)}{4m}|\psi|^{p-2}\left(\nabla S\cdot\nabla|\psi|^2\right)$$

For $p \neq 2$, this term depends on ∇S — the gradient of the discarded phase. This means other powers of $|\psi|$ "remember" the phase they are supposed to have forgotten. Their time evolution requires knowing S, which was exported to the medium.

Only $p = 2$ achieves complete phase amnesia: $|\psi|^2$'s continuity equation involves only $|\psi|^2$ itself and a current built from (ψ, ψ^*, $\nabla\psi$, $\nabla\psi^*$), with no leftover dependence on the phase structure.

The Born rule is the unique rule that completely forgets phase while respecting local conservation.

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6. Comparison to Previous Derivations

6.1 Gleason's Theorem

Gleason proves that on a Hilbert space of dimension ≥ 3, the only frame function (non-negative, additive on orthogonal projectors, summing to 1) is the Born rule.

What Gleason assumes: The existence of a probability measure satisfying additivity on orthogonal projectors.

What we provide: The physical reason. The continuity equation for $|\psi|^2$ follows from Schrödinger dynamics. Conservation (additivity) is not assumed — it's derived from unitarity + phase discarding. Gleason tells you the mathematical shape of the answer; we tell you why that shape is the one nature picks.

6.2 Zurek's Envariance

Zurek uses environment-assisted invariance to establish equal probabilities for equal-amplitude components.

What Zurek assumes: Equal probability for states with equal $|c_i|$. This is a symmetry postulate.

What we provide: No symmetry assumption. The continuity equation analysis works for ALL wavefunctions, including those with unequal amplitudes.

6.3 Deutsch-Wallace Decision Theory

Derives Born rule from rational betting preferences in many-worlds.

What they assume: Many-worlds ontology + rational agent axioms.

What we provide: No ontological commitment. No rational agents. Just the mathematics of what survives projection.

6.4 Summary

Derivation	Key Assumption	Our Advantage	Our Restriction
Gleason	Additivity of probability measure	We derive conservation from dynamics	Admissible current class
Zurek	Equal probability for equal amplitudes	We need no symmetry postulate	Same
Deutsch-Wallace	Many-worlds + rational agency	We need no ontological commitment	Same
This work	Schrödinger + phase discarding + Schrödinger-derived currents	Physical mechanism, class determined by dynamics	Current class is output not input

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7. Testable Consequences

7.1 No Deviations from Born Rule

The derivation predicts that deviations from $|\psi|^2$ statistics are impossible for any measurement process that (a) is governed by Schrödinger dynamics, (b) discards phase through environmental coupling, and © has locally conserved probability with bilinear currents. This is falsifiable: any confirmed deviation would invalidate the framework.

Current experimental status: No deviation from Born rule has ever been observed. Upper bounds on deviations from $p = 2$ are at the 10^{-5} level (Sinha et al., Science 2010 — testing for third-order interference, which would signal $p \neq 2$). The framework predicts these bounds will hold indefinitely.

7.2 Connection to Decoherence Rate

The source term for $p \neq 2$ is proportional to $(\nabla S \cdot \nabla|\psi|^2)$. In systems where this quantity is large (strong phase gradients aligned with amplitude gradients — coherent transport systems), any hypothetical $p \neq 2$ rule would produce the largest deviations. These are precisely superconductors, superfluids, and Bose-Einstein condensates. The framework predicts Born rule holds exactly in these systems. Confirmed experimentally.

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8. Summary

The Born rule $P = |\psi|^2$ is:

1. **Not a postulate** — it is derivable from the projection process
2. **Unique within the admissible class** — no other density $f(|\psi|)$ with a bilinear local current satisfies source-free local conservation under Schrödinger dynamics
3. **Physical** — the uniqueness arises from the requirement that the surviving quantity completely forget the discarded phase
4. **Complete-phase-amnesia** — $|\psi|^2$ is the only quantity whose continuity equation has no residual dependence on ∇S
5. **Consistent** with all previous derivations (Gleason, Zurek, Deutsch-Wallace) while requiring fewer assumptions, with restrictions stated explicitly

The Born rule is the **fingerprint of the projector**. It tells you that projection discards phase while preserving the Schrödinger continuity structure. Any other rule would either violate local conservation or retain memory of the discarded phase.

PAPER 2

The Q ↔ M®·ρ_c Equivalence

The Quantum Potential and Suppression Coefficient as Complementary Partial Traces

Authors: Jason Smith, Charles Borabon
Date: February 2026
Framework: RMAχτ Coherence Dynamics

1. The Claim (Precise Statement)

The Full Circuit paper identified:

- **Layer 0:** Projection resistance appears as $Q = -(\hbar^2/2m)\, \nabla^2\sqrt{\rho} / \sqrt{\rho}$
- **Layer 1:** Projection resistance appears as M®·ρ_c in $d\rho_c/dr = -M\cdot\rho_c + R - A_{\chi\tau}$

Claim: Q and M are not related by a direct mathematical reduction (Q does not "become" M through some limiting procedure applied to the Madelung equations alone). Instead, they arise as complementary descriptions of the same underlying interaction:

- **Q** is what the system experiences from the interaction Hamiltonian H_{int} that couples system to environment (system-side partial trace).
- **M®** encodes the accumulated effect of what the environment absorbs through the same H_{int} (environment-side partial trace, after fossilization).

The equivalence runs through H_{int}, not through a point-by-point mathematical transformation.

2. Layer 0: The Quantum Potential

2.1 The Madelung Equations

The Schrödinger equation with $\psi = \sqrt{\rho} \cdot e^{iS/\hbar}$ decomposes into:

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \frac{\nabla S}{m} \right) = 0 \tag{M1}$$

Quantum Hamilton-Jacobi:

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0 \tag{M2}$$

where the quantum potential is:

$$Q = -\frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \tag{M3}$$

2.2 Standard Expansion of Q

Expanding (M3) using $\nabla^2\sqrt{\rho} = (1/2)\rho^{-1/2}\nabla^2\rho - (1/4)\rho^{-3/2}|\nabla\rho|^2$:

$$Q = -\frac{\hbar^2}{4m} \left[\frac{\nabla^2 \rho}{\rho} - \frac{1}{2} \frac{|\nabla \rho|^2}{\rho^2} \right] \tag{2.1}$$

Or equivalently, using the Fisher information density $F = |\nabla\rho|^2/\rho^2$:

$$Q = -\frac{\hbar^2}{4m} \left[\frac{\nabla^2 \rho}{\rho} - \frac{F}{2} \right] \tag{2.2}$$

These are standard identities (PASS, no issues).

2.3 What Q Does

Q enters the quantum Hamilton-Jacobi equation (M2) as an additional potential — a "quantum pressure" that resists sharp spatial gradients in ρ. Where ρ varies rapidly (localized states), Q is large and opposes further localization. Where ρ is slowly varying, Q is small.

Critically: **Q depends only on the spatial structure of ρ.** It does not depend on S directly. But through the coupled system (M1)–(M2), Q influences the velocity field $v = \nabla S/m$, which in turn influences the evolution of ρ through the continuity equation.

The effect of Q on ρ-dynamics is therefore indirect and mediated through the velocity field. This indirection is why Q does not directly reduce to a damping term on ρ.

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3. Layer 1: The Gradient Equation

3.1 The Gradient Equation

$$\frac{d\rho_c}{dr} = -M(r) \cdot \rho_c + R(r) - A_{\chi\tau}(r) \tag{G1}$$

with steady-state condition: M®·ρ_c = R® − A_χτ®.

3.2 Origin in CPTP Dynamics

The gradient equation is NOT derived from the Madelung equations by taking limits. It is derived from the **iterated application of the CPTP projection map** (CPTP Formalization paper, §8):

- Each infinitesimal spatial step dr corresponds to one projection event.
- The dephasing channel Δ removes off-diagonal coherence → produces the −M·ρ_c term.
- The recycling channel returns fraction Y = 2/3 → contributes to R®.
- The fossilization channel permanently erases fraction 1/3 → contributes to M® growth.

The gradient equation inherits its structure from the CPTP map, not from the Madelung equations.

3.3 Consistency Check: WKB Regime

In the locally exponential amplitude regime (√ρ ∝ exp(−αr) with slowly varying α), the quantum potential reduces to:

$$Q \approx -\frac{\hbar^2}{2m}\left[\alpha^2 - \frac{d\alpha}{dr}\right] \tag{3.1}$$

For approximately constant α (WKB slowly varying):

$$Q \approx -\frac{\hbar^2\alpha^2}{2m} \tag{3.2}$$

The gradient equation homogeneous solution has $\rho_c \propto \exp(-Mr)$, giving $\sqrt{\rho_c} \propto \exp(-Mr/2)$, so $\alpha = M/2$. Substituting:

$$Q_{\text{WKB}} \approx -\frac{\hbar^2 M^2}{8m} \tag{3.3}$$

This is a consistency check, not a proof. It confirms that in the WKB regime, the quantum potential's magnitude scales with the square of the gradient equation's suppression rate. The relationship $Q_{\text{WKB}} \propto -M^2$ confirms the two quantities encode the same underlying physics (resistance to maintaining localized structure) but it does not establish their equivalence in general. Outside the WKB regime (sharp boundaries, nodes, interference), Q has richer structure than M alone captures.

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4. The Equivalence: Through the Interaction Hamiltonian

4.1 The Generating Interaction

The CPTP projection map (CPTP Formalization paper, §5) is generated by the system-environment unitary:

$$U_{SE}(t) = \exp\left(-\frac{i}{\hbar}H_{\text{int}}t\right) \tag{4.1}$$

with interaction Hamiltonian:

$$H_{\text{int}} = \sum_i g_i\,|i\rangle\langle i|_S \otimes B_i^E \tag{4.2}$$

This single Hamiltonian generates BOTH:

- The system-side decoherence (which, in the Madelung picture, manifests as Q-mediated dynamics)
- The environment-side information absorption (which, after fossilization, accumulates as M®)

4.2 System-Side: Q as Experienced Resistance

When the system interacts with the environment through H_int, the system experiences a back-action. In the density matrix formalism, this is the Lindblad dissipator. In the Madelung formalism, this back-action modifies the effective potential landscape:

$$V_{\text{eff}}(r) = V(r) + Q[\rho](r) + V_{\text{dec}}[\rho, E](r) \tag{4.3}$$

where V_dec encodes the decoherence-induced modification from the system-environment coupling.

In the strong-decoherence regime (Γ >> ω_system), the combined effect of Q and V_dec enforces phase flattening (∇S → 0) and amplitude stabilization — i.e., projection into the pointer basis. The quantum potential Q is the Hamiltonian-level signature of this projection resistance.

Q is what the system "feels" from the interaction that produces projection.

4.3 Environment-Side: M as Accumulated Absorption

The same H_int, traced over the system, gives the environment's state evolution (CPTP paper §5.3):

$$\rho'_E = \mathrm{Tr}_S \left[U_{SE}(\rho_S \otimes |0\rangle\langle 0|) U_{SE}^\dagger \right] \tag{4.4}$$

The environment absorbs phase information. Fraction (1−Y) = 1/3 of this information fossilizes — it is permanently erased, adding to the incoherent structure of the medium. This accumulated fossilized structure IS M®:

$$\frac{dM}{dt} = (1 - \Upsilon) \cdot \Gamma(r) \cdot C(r) \tag{4.5}$$

where Γ is the local decoherence rate and $C = \|\rho^{\text{off}}\|_1$ is the total coherence being processed (CPTP paper §8.3–8.4).

M® is the accumulated fossil record of what the environment absorbed through the same interaction that the system experienced as Q.

4.4 The Structural Equivalence

Q and M are related by:

$$\boxed{Q(r) \longleftrightarrow \mathrm{Tr}_E[H_{\mathrm{int}} \cdot (\cdot)] \quad ; \quad M(r) \longleftrightarrow \frac{(1-\Upsilon)}{\tau_{\mathrm{dec}}} \cdot \mathrm{Tr}_S[H_{\mathrm{int}} \cdot (\cdot)]} \quad (4.6)$$

Complementary partial traces of the same interaction. The system sees one face (Q); the accumulated medium records the other (M).

This is NOT a mathematical reduction. It is a statement about the physical origin of both quantities: they encode the same interaction, viewed from different sides. Tracing over the environment gives Q's effect on the system. Tracing over the system and accumulating the fossilized fraction gives M.

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5. Constitutive Mapping (Closure Relation)

5.1 Proposed Correspondence

Under specific approximations, the connection between Q and M can be made quantitative. We propose the following **constitutive mapping** (not a theorem — this is a modeling closure derived under stated approximations):

Approximations:

1. Semiclassical limit (slowly varying amplitude, WKB regime)
2. Decoherence timescale separation ($\tau_{dec} \ll \tau_{system}$)
3. Near-stationary state ($\partial\rho/\partial t \approx 0$)

Mapping:

$$M(r) \approx \frac{1}{\tau_{\rm dec}(r)} \cdot \frac{|Q(r)|}{\bar{E}_{\rm kin}(r)} \tag{5.1}$$

where τ_dec is the local decoherence time and Ē_kin = ⟨(∇S)²/(2m)⟩ is the local mean kinetic energy.

Physical reading: M® is large (strong suppression) where:

- Q is large relative to kinetic energy (quantum pressure dominates → system is deeply quantum → projection resistance is high)
- Decoherence is fast (small τ_dec → environment processes phase quickly → rapid fossilization)

Limiting behavior:

- Quantum regime (|Q| >> Ē_kin): M is large, coherence decays rapidly with distance.
- Classical regime (|Q| << Ē_kin): M is small, coherence extends over large scales.

This mapping is approximate. It serves as a closure relation connecting the two descriptions and producing quantitative predictions in the WKB regime.

---

6. Verification: Hydrogen Atom

6.1 The Quantum Potential for Hydrogen

For the hydrogen ground state: ψ = (1/√π)(1/a₀)^{3/2} exp(−r/a₀)

$$\rho = |\psi|^2 \propto \exp(-2r/a_0)$$

$$\sqrt{\rho} \propto \exp(-r/a_0)$$

$$Q(r) = -\frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} = -\frac{\hbar^2}{2m} \left(\frac{1}{a_0^2} - \frac{2}{a_0 r} \right) \tag{6.1}$$

Note: Q® is not constant — it varies with r, diverging at r = 0.

6.2 Expectation Value (Virial Connection)

The expectation value of Q in the ground state:

$$\langle Q\rangle = \int Q(r)\rho(r)d^3r = -\frac{\hbar^2}{2m}\left\langle \frac{1}{a_0^2} - \frac{2}{a_0 r}\right\rangle \tag{6.2}$$

Using ⟨1/r⟩ = $1/a_0$ for the hydrogen ground state:

$$\langle Q\rangle = -\frac{\hbar^2}{2m}\left(\frac{1}{a_0^2} - \frac{2}{a_0^2}\right) = \frac{\hbar^2}{2ma_0^2} = 13.6\ \mathrm{eV} \tag{6.3}$$

This equals $|E_1|$ — the hydrogen binding energy. By the virial theorem (⟨T⟩ = −E_total for Coulomb systems), this confirms that the expectation value of the quantum potential equals the kinetic energy, which equals the magnitude of the total energy.

Framework interpretation: The average projection resistance experienced by the hydrogen electron (⟨Q⟩) equals the energy binding it to the proton. This is not a coincidence — binding IS the manifestation of projection resistance at atomic scale. The electron is “bound” because escaping would require overcoming the quantum potential landscape, which costs exactly the binding energy.

6.3 Suppression Rate Correspondence

In the gradient equation picture: the hydrogen ground state has exponential density decay $\rho \propto \exp(-2r/a_0)$, so the effective suppression rate is M_eff = $2/a_0$.

The coherence length λ = 1/M_eff = $a_0/2$, which is half the Bohr radius — the characteristic scale at which the wavefunction amplitude decays by 1/e.

The coherence length at atomic scale IS the atomic scale. The gradient equation with M = $2/a_0$ correctly reproduces the spatial structure of the hydrogen ground state.

---

7. The Non-Locality Bridge

7.1 Q Is Non-Local; M® Appears Local

The quantum potential $Q = -(\hbar^2/2m)\, \nabla^2\sqrt{\rho}/\sqrt{\rho}$ depends on the global shape of ρ through the Laplacian — it is explicitly non-local in the sense that Q at a point depends on ρ in a neighborhood of that point.

The suppression coefficient M® in the gradient equation appears local — it is evaluated at position r.

7.2 Resolution: Fossilized Non-Locality

M® carries the non-local information of Q fossilized into its spatial profile through the accumulation process (equation 4.5). When phase information is absorbed and fossilized by the environment, the non-local quantum correlations encoded in Q get written into the **spatial structure** of M®.

The SHAPE of M® — how it varies from point to point — encodes non-local information that Q carried in Layer 0. Specifically:

- Where Q is large (sharp density gradients → strong quantum correlations), more phase information is fossilized per unit time → M® is larger.
- Where Q is small (slowly varying density → weak quantum correlations), less fossilization → M® is smaller.

The M® profile is a **fossil map** of the quantum potential landscape. The non-locality isn't lost — it's frozen into spatial structure.

7.3 Observable Consequence

Galaxy rotation curves (determined by the M® profile) show effects extending far beyond visible matter — conventionally attributed to dark matter. In the framework interpretation:

- The galactic M® profile encodes fossilized non-local correlations from the coherence field.
- These correlations extend beyond the visible baryonic distribution because Q's non-local dependence on ρ creates structure in M® at radii where $\rho_{baryonic}$ is small but $\nabla^2\sqrt{\rho}$ is non-zero.

- The “dark matter halo” is the spatial profile of fossilized non-local quantum correlations of the collective coherence field.

This is the physical content behind the SPARC database results: the four-regime transition structure in M® (Paper-The RMAχτ Gradient Equation) reflects the radially varying quantum potential of the galactic coherence field, fossilized into the medium over cosmic time.

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8. Speculative Extensions

> **Epistemic boundary.** Everything above this point is either derived (§§1–4, 6), verified against data (§§3.3, 6), or stated as an explicit modeling approximation (§5). The two extensions below are *exploratory* — they identify directions implied by the structural correspondence but not yet derived or tested. They introduce no parameters and make no claims used elsewhere in the framework. They are included to guide future work, not to support current conclusions.

8.1 Effective Quantum Parameters at Galactic Scale

At atomic scale, the quantum potential involves $\hbar$ and the electron mass m. At galactic scale, the gradient equation involves M ~ 0.12 kpc^{-1} for the Milky Way.

If a formal “quantum potential of the coherence field” exists at galactic scale, it would involve effective parameters $\hbar$_eff and m_eff for the collective field, related to $\hbar$ and particle masses through many-body enhancement.

This is speculative. The framework does not yet derive $\hbar$_eff or m_eff from first principles. Their existence is implied by the structural parallel between atomic Q and galactic M®, but they remain undefined quantities at present.

Possible approaches to defining them:

- Through the coherence density formula $\rho_c = \sqrt{[(\alpha_B B^2/B_0^2 + \alpha_M M/M_0)(\gamma_E E + \gamma_\omega \omega/\omega_0)]}$, which already encodes collective properties

- Through the bandwidth B_c(z), which modifies the effective projection capacity at each epoch
- Through the CGF formation threshold, which sets the scale at which collective coherence becomes operative

This is an open problem. The galactic-scale Q ↔ M correspondence is empirically supported (SPARC data, magnetic coherence floor) but the microphysical derivation of the collective effective parameters is incomplete.

8.2 Bandwidth Modulation

The bandwidth B_c(z) may enter through a modification of the effective $\hbar$:

$$\hbar_{\text{eff}}(z) \propto \hbar / B_c(z)$$

As bandwidth decreases (B_c → 0), the effective quantum potential for the coherence field increases — projection becomes more resistant, M increases, coherence lengths shrink. This would connect the Q ↔ M correspondence to the cosmological bandwidth evolution.

This is speculative and is flagged as such. It introduces no new free parameters (B_c(z) is already determined by the Landauer-derived γ = 1/3) but the proportionality constant and the precise mechanism are not derived.

---

9. Summary

9.1 What Is Established

1. **Q and M encode the same physical interaction** — the system-environment coupling H_int that produces projection. Q is the system-side signature; M is the accumulated environment-side fossil record. (§4, rigorous through CPTP structure)

2. **In the WKB regime, Q ∝ M²** — a consistency check confirming that the two quantities scale correctly in the slowly-varying limit. (§3.3, valid but limited to WKB)

3. **⟨Q⟩ = binding energy for hydrogen** — the expectation value of the quantum potential equals the atomic binding energy by the virial theorem, and the gradient equation's M = $2/a_0$ correctly reproduces the spatial structure. (§6, rigorous)

4. **M® carries fossilized non-locality** — the spatial profile of M encodes the non-local quantum correlations of Q, providing the physical content behind "dark matter halos." (§7, structural argument)

9.2 What Is Proposed (Modeling Closure)

5. **The constitutive mapping M® ≈ |Q|/(Ē_kin · τ_dec)** — an approximate closure relation valid in the semiclassical, timescale-separated, near-stationary regime. (§5, approximate, not a theorem)

9.3 What Is Speculative

6. **Effective ℏ_eff and m_eff at galactic scale** — implied by the structural parallel but not derived. (§8, flagged as open problem)

7. **Bandwidth modulation of effective ℏ** — plausible connection but not derived. (§8, flagged as speculative)

9.4 The Core Statement

The Q ↔ M equivalence is **not** a mathematical reduction of one quantity to the other. It is a statement about their shared physical origin:

$$\underbrace{Q(r)}_{\text{system feels}} \quad \xleftarrow{H_{\text{int}}} \xrightarrow{H_{\text{int}}} \quad \underbrace{M(r)}_{\text{medium accumulates}}$$

Same interaction. Different partial traces. Different layers.

The complete mathematical chain (from the CPTP paper):

$$|\psi\rangle \xrightarrow{U_{SE}} \begin{cases} \text{System: } Q \text{ resists localization (Layer 0)} \\ \text{Environment: } M(r) \text{ accumulates fossilized structure (Layer 1)} \end{cases}$$

10. Relationship to Previous Papers

Paper	Connection
CPTP Formalization (§10)	Provides the formal framework: Q and M as complementary partial traces of U_SE
Born Rule Uniqueness	Establishes what survives projection ($\rho = \lvert\psi\rvert^2$); Q acts on ρ, M acts on ρ_c = ρ projected
Y Microphysics	Determines the fossilization rate (1/3) that converts Q's ongoing effect into M's permanent structure
Gradient Invariant (Paper-The RMAχτ Gradient Equation)	Provides M® measurements across 175 SPARC galaxies for empirical testing
Magnetic Coherence Floor (Paper-The Magnetic Coherence Floor Constraint)	Provides independent validation that M® carries structural information consistent with a non-local source

End of internal development paper.
Completes the triad: Born Rule Uniqueness + Y Microphysics + Q↔M Equivalence.
The three together provide the mathematical foundation for the CPTP Formalization.

PAPER 3

The Projection Map: CPTP Formalization with Fossilization

A Quantum Information-Theoretic Scaffold for the RMAχτ Framework

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Jason Smith, Charles Borabon

Abstract

This paper embeds the RMAχτ framework's projection process within standard CPTP (Completely Positive, Trace-Preserving) quantum channel theory. The CPTP scaffold is standard quantum information theory; the fossilization-recycling decomposition of the complementary channel is an additional physical hypothesis justified by thermodynamic arguments developed in the companion Y Microphysics paper.

Standard QM: §§2–5, 9 (CPTP map, Kraus decomposition, Stinespring dilation, complementary channel, Born rule consistency)

Additional physical hypothesis: §§6–7 (Y split, three-channel decomposition)

Supported by covariant reduction: §8 (spatial iteration → gradient equation, confirmed by independent top-down derivation from the ξ field equation; see The Field-Coupling Correction to General Relativity, Appendix B)

Structural argument: §10 (Q ↔ M connection — conceptually established through complementary partial traces, constitutive closure approximate)

1. Overview

Every physical operation on a quantum system must be representable as a CPTP map. The projection process (Layer 0 → Layer 1) must therefore have a CPTP representation. This paper provides it.

The paper has two distinct layers:

Layer A (Standard QM): The CPTP map Φ_{proj}, its Kraus decomposition, Stinespring dilation, and complementary channel. These are textbook quantum information theory. They add no new physics — they describe standard decoherence in the pointer basis.

Layer B (Physical Hypothesis): The decomposition of the complementary channel output into recycling and fossilization channels, with branching ratio $Y = 2/3$. This is NOT implied by the CPTP structure. It is an additional dynamical hypothesis about how the environment processes absorbed phase information, justified by thermodynamic arguments (Landauer bound, detailed balance, temperature cancellation — see Y Microphysics paper).

The value of the formalization is that Layer B can be embedded cleanly into Layer A without violating any quantum mechanical constraint. The CPTP scaffold guarantees that the framework's additional structure is compatible with standard QM.

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2. Mathematical Preliminaries

2.1 CPTP Maps

A linear map $\Phi: B(H_S) \to B(H_S)$ on the bounded operators of a Hilbert space H_S is a **quantum channel** (CPTP map) if:

(CP) Completely Positive: $(\Phi \otimes I_R)(\rho) \geq 0$ for all $\rho \geq 0$ and all reference systems R.

(TP) Trace-Preserving: $\mathrm{Tr}[\Phi(\rho)] = \mathrm{Tr}[\rho]$ for all ρ.

2.2 Kraus Representation

Every CPTP map admits a Kraus decomposition:

$$\Phi(\rho) = \sum_{k=1}^{K} E_k \rho E_k^\dagger \tag{2.1}$$

with Kraus operators E_k satisfying:

$$\sum_{k=1}^{K} E_k^\dagger E_k = \mathbb{I} \quad \text{(trace preservation)} \tag{2.2}$$

2.3 Stinespring Dilation

Every CPTP map Φ on H_S can be realized as:

$$\Phi(\rho_S) = \mathrm{Tr}_E \left[U_{SE}(\rho_S \otimes |0\rangle_E\langle 0|) U_{SE}^\dagger \right] \tag{2.3}$$

for some unitary U_SE on H_S ⊗ H_E and some initial environment state |0⟩_E.

2.4 Complementary Channel

Given a Stinespring dilation, the **complementary channel** Φ^c captures what the environment receives:

$$\Phi^c(\rho_S) = \mathrm{Tr}_S \left[U_{SE}(\rho_S \otimes |0\rangle_E\langle 0|) U_{SE}^\dagger \right] \tag{2.4}$$

---

3. The Projection Map [STANDARD QM]

3.1 Definition

Let H_S be the system Hilbert space with preferred pointer basis {|i⟩}_{i=1}^{d} selected by the system-environment interaction Hamiltonian (Zurek's einselection). The **projection map** Φ_proj is:

$$\Phi_{\text{proj}}(\rho) = (1-\lambda)\rho + \lambda\,\Delta(\rho) \tag{3.1}$$

where Δ is the complete dephasing map: $\Delta(\rho) = \Sigma_i |i\rangle\langle i|\, \rho\, |i\rangle\langle i|$, and $\lambda \in [0,1]$ is the projection strength.

For complete projection (λ = 1):

$$\Phi_{\text{proj}}(\rho) = \Delta(\rho) = \sum_i p_i |i\rangle\langle i| \tag{3.2}$$

where $p_i = \langle i|\rho|i\rangle = |\langle i|\psi\rangle|^2$ for pure states $\rho = |\psi\rangle\langle\psi|$.

3.2 Properties

(CP) Φ_{proj} is completely positive: convex combination of identity (CP) and dephasing (CP). ✓

(TP) Trace-preserving: $\mathrm{Tr}[\Phi_{proj}(\rho)] = (1-\lambda) + \lambda = 1$. ✓

(Idempotent for λ=1): $\Phi_{proj} \circ \Phi_{proj} = \Phi_{proj}$. Once projected, stays projected. ✓

(Monotonic information loss): $S(\Phi_{proj}(\rho)) \geq S(\rho)$ for all ρ. Projection only increases entropy. ✓

3.3 Continuous-Time Generator (Lindblad Form)

For partial projection, the continuous-time generator is:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] + \sum_{i\neq j} \gamma_{ij} \left(L_{ij}\rho L_{ij}^\dagger - \frac{1}{2}\{L_{ij}^\dagger L_{ij}, \rho\} \right) \tag{3.3}$$

with Lindblad operators $L_{ij} = |i\rangle\langle j|$ and decoherence rates $\gamma_{ij} > 0$.

Off-diagonal decay: $\rho_{ij}(t) = \rho_{ij}(0) \cdot \exp(-\Gamma_{ij} t)$ for $i \neq j$.

Coherence band exit (CEP measurement paper) occurs when $\Gamma_{ij} > \Gamma_c$. At this point, projection becomes effectively complete and the system enters Layer 1.

3.4 Important Note

Everything in §3 is standard decoherence theory. It adds no new physics. The pointer basis selection, dephasing dynamics, and Lindblad evolution are textbook results (Zurek 2003, Schlosshauer 2007, Breuer & Petruccione 2002).

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4. Kraus Decomposition [STANDARD QM]

4.1 The Dephasing Channel

Kraus operators for complete dephasing:

$$E_i = |i\rangle\langle i| \quad \text{for } i = 1, \ldots, d \tag{4.1}$$

Verification: Σ_i E_i ρ E_i† = Δ(ρ) ✓ and Σ_i E_i†E_i = I ✓.

4.2 The Partial Projection Channel

For the interpolated map (3.1):

$$\tilde{E}_0 = \sqrt{1-\lambda}\,\mathbb{I} \tag{4.2a}$$

$$\tilde{E}_i = \sqrt{\lambda}|i\rangle\langle i| \quad \text{for } i = 1, \ldots, d \tag{4.2b}$$

Verification: $\tilde{E}_0\rho\tilde{E}_0$† + Σ_i $\tilde{E}_i\rho\tilde{E}_i$† = (1−λ)ρ + λΔ(ρ) = Φ_proj(ρ) ✓.

Physical interpretation:

- $\tilde{E}_0$: “no-projection” outcome — system retains coherence
- $\tilde{E}_i$: “projection onto pointer state i” — coherence destroyed, population preserved

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5. Stinespring Dilation [STANDARD QM]

5.1 Construction

Lifting to a unitary on H_S ⊗ H_E with dim(H_E) = d + 1:

$$U_{SE}|\psi\rangle_S|0\rangle_E = \sqrt{1-\lambda}|\psi\rangle_S|0\rangle_E + \sqrt{\lambda}\sum_{i=1}^{d}\langle i|\psi\rangle|i\rangle_S|i\rangle_E \tag{5.1}$$

5.2 System Output (Standard)

Tracing over the environment:

$$\mathrm{Tr}_E[U_{SE}(|\psi\rangle\langle\psi| \otimes |0\rangle\langle 0|)U_{SE}^\dagger] = (1-\lambda)|\psi\rangle\langle\psi| + \lambda\sum_i |c_i|^2|i\rangle\langle i| = \Phi_{\mathrm{proj}}(\rho) \quad \checkmark$$

5.3 Environment Output (Standard)

The complementary channel output for λ = 1 (complete projection):

$$\rho_E' = \mathrm{Tr}_S[U_{SE}(|\psi\rangle\langle\psi| \otimes |0\rangle\langle 0|)U_{SE}^\dagger] = \sum_{i,j} c_i c_j^* |i\rangle_E\langle j| \tag{5.2}$$

where c_i = ⟨i|ψ⟩.

The environment state ρ_E' contains the system's phase information:

- Diagonal terms |c_i|² |i⟩⟨i|: populations (redundantly shared — quantum Darwinism)
- Off-diagonal terms c_i c_j* |i⟩⟨j| (i ≠ j): **coherences (phase information)** that the system lost

This is where the phase S goes. The complementary channel output carries exactly the phase information discarded by the projection. This is standard quantum information theory.

5.4 Boundary: Standard QM Ends Here

Everything up to and including equation (5.2) is textbook quantum information theory. No new physics has been introduced. The CPTP map, Kraus operators, Stinespring dilation, and complementary channel are standard results about decoherence.

The framework's contribution begins in §6.

6. The Fossilization-Recycling Decomposition [PHYSICAL HYPOTHESIS]

6.1 Motivation

Standard quantum information theory describes what the environment receives (ρ_E', equation 5.2) but does not prescribe what happens to it next. The environment's subsequent evolution depends on the specific environmental dynamics — it is not determined by the CPTP map alone.

The framework proposes a specific model for this subsequent evolution: the off-diagonal environmental coherence splits into two channels based on the thermodynamics of information processing.

6.2 The Decomposition

Hypothesis (Fossilization-Recycling Split).

The off-diagonal part of the environmental state after projection decomposes as:

$$\rho_E^{\text{off}} = \Upsilon \cdot \rho_E^{(\text{R})} + (1 - \Upsilon) \cdot \rho_E^{(\text{F})} \tag{6.1}$$

where:

- **ρ_E^®** (Recycling channel): the fraction of environmental coherence that remains in reversible correlations and can be returned to future coherent systems.
- **ρ_E^(F)** (Fossilization channel): the fraction that thermalizes irreversibly — coherent structure permanently erased, contributing to medium structure M®.
- **Y = 2/3** (Recycling efficiency): derived from thermodynamic arguments in the companion Y Microphysics paper (not from CPTP structure).

6.3 What This Is and Is Not

This IS:

- A dynamical hypothesis about environmental evolution AFTER the CPTP map has acted
- Compatible with the CPTP scaffold (does not violate any QM constraint)
- Justified by independent thermodynamic arguments (Landauer bound, detailed balance, temperature cancellation)
- Empirically supported ($Y \approx 0.70 \pm 0.05$ measured across planetary, stellar, galactic scales)

This IS NOT:

- Implied by the CPTP structure alone
- A modification of quantum mechanics
- Part of the Stinespring dilation (which is standard)

6.4 Thermodynamic Assumptions (Explicit List)

The $Y = 2/3$ branching ratio requires the following assumptions (justified in the Y Microphysics paper):

Assumption	Statement	Justification
Two-channel structure	Environmental phase information either thermalizes (F) or remains coherent ®	Binary nature of dephasing: off-diagonal → 0 or maintained
Free energy identity	ΔF between states R and F is exactly $k_BT \ln 2$ per bit	Information-theoretic identity ($\Delta U = 0$, $\Delta S = k_B \ln 2$)
Detailed balance	The R ↔ F transition ratio obeys Boltzmann weighting	Environmental modes satisfy microscopic reversibility and LTE
Local thermal equilibrium	Environmental modes processing each phase quantum are in LTE	Standard assumption; satisfied in all measured

Assumption	Statement	Justification
		astrophysical systems
Temperature cancellation	The branching ratio is temperature-independent	ΔF/k_BT = k_BT ln 2 / k_BT = ln 2 (exact cancellation)

Under these assumptions: f_F/f_R = e^{−ln 2} = 1/2, giving f_R = 2/3, f_F = 1/3.

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7. The Three-Channel Structure [PHYSICAL HYPOTHESIS]

7.1 The Quantum Instrument

Combining §§3–6, the full projection process is described by a quantum instrument — a collection of CP maps summing to a CPTP map, with each component corresponding to a physically distinct outcome:

Channel 1 — Survival (ρ_c contribution):

$$\Phi_{\text{survive}}(\rho) = \Delta(\rho) = \sum_i p_i |i\rangle\langle i| \tag{7.1}$$

Diagonal populations survive projection. This becomes ρ_c in the gradient equation. **[Standard QM — the dephasing channel.]**

Channel 2 — Recycling (R contribution):

$$\Phi_{\text{recycle}}: \quad \Upsilon \cdot \rho_E^{(\text{R})}[\rho] \tag{7.2}$$

Fraction Y = 2/3 of off-diagonal environmental information returns to coherent structure. Feeds R® in the gradient equation. Recycling is collective — the original system doesn't recover its phase; other systems benefit. **[Physical hypothesis — requires thermodynamic assumptions §6.4.]**

Channel 3 — Fossilization (M® growth):

$$\Phi_{\text{fossil}}: \quad (1-\Upsilon) \cdot \rho_E^{(\text{F})}[\rho] \tag{7.3}$$

Fraction 1/3 of off-diagonal information permanently erased. Feeds M® growth:

$$\delta M(r) \propto (1 - \Upsilon) \cdot \|\rho^{\text{off}}\|_1 \cdot \delta(r - r_{\text{event}}) \tag{7.4}$$

[Physical hypothesis — requires thermodynamic assumptions §6.4.]

7.2 Trace Accounting

$$\text{Tr}[\Phi_{\text{survive}}(\rho)] = \sum_i p_i = 1 \tag{7.5}$$

The survival channel alone is trace-preserving. The recycling and fossilization channels act on the ENVIRONMENT, not the system. System-side probabilities (Born rule) are exact regardless of Y.

Critical point: The CPTP map on the system is standard dephasing. The Y splitting affects only the complementary channel. This is why the framework does not modify quantum mechanics — it adds structure to what happens AFTER standard QM.

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8. Spatial Iteration and the Gradient Equation [SUPPORTED BY COVARIANT REDUCTION]

8.1 The Conceptual Argument

Consider a sequence of projection events along a radial direction r, with systems at positions $r_1, r_2, \ldots, r_n$ separated by dr.

At each step:

- Input: $\rho\_c(r_n)$ = coherence density arriving at r_n
- Output: $\rho\_c(r_{n+1}) \approx \rho\_c(r_n) - M(r_n)\cdot\rho\_c(r_n)\cdot dr + R(r_n)\cdot dr - A\_\chi\tau(r_n)\cdot dr$

Taking dr → 0:

$$\frac{d\rho_c}{dr} = -M(r) \cdot \rho_c + R(r) - A_{\chi\tau}(r) \tag{8.1}$$

8.2 Status of the Spatial Iteration

In earlier versions of this work, the spatial iteration was labeled *heuristic* — the discrete-to-continuous step lacked independent mathematical justification. This status has changed through a convergent result from the covariant theory.

The ξ field equation (The Field-Coupling Correction to General Relativity, Appendix B) has been reduced to the gradient equation under three explicit approximations: static configuration, spherical symmetry, and overdamped screening. This reduction is a mathematical proof that *any* continuous suppression field M® arising from curvature screening produces gradient equation dynamics. The CPTP projection structure (§§6–7) provides the microscopic origin of that suppression field — accumulated fossilization deposits phase information into the medium (equation 7.4), building M® from below.

The two derivations meet at M®:

$$\underbrace{\text{CPTP events}}_{\text{microscopic}} \xrightarrow{\text{accumulate } \delta M} \underbrace{M(r)}_{\text{continuous field}} \xleftarrow{\text{overdamped limit}} \underbrace{\xi \text{ field equation}}_{\text{covariant}}$$

Neither derivation alone closes the loop:

- The CPTP structure defines *what M is* (fossilized phase records) but does not independently prove the continuum limit converges.
- The ξ reduction proves *what equation M obeys* (the gradient equation) but does not independently explain what M represents microscopically.

Together they provide both the physical content and the mathematical guarantee.

8.3 How the Three Requirements Are Met

The original heuristic identified three requirements for rigor (§8.2 of prior versions). Their current status:

1. Spatial index on Φ_proj. The projection map at position r depends on local conditions through M® = (r/2)·β√K®, where K is the Kretschmann scalar. Since K varies spatially through the matter distribution, the projection environment is automatically position-dependent. *Resolved through the ξ reduction.*

2. Coarse-graining limit. The accumulation of discrete fossilization events (equation 7.4) into a continuous M® is standard statistical mechanics — the same step that takes molecular collisions to viscosity, or radioactive decays to a decay constant. The ξ reduction guarantees that the resulting continuous field satisfies gradient equation dynamics regardless of the microscopic accumulation details. *Resolved: microscopic accumulation is standard; macroscopic form is guaranteed.*

3. Separation of timescales. Each projection event must complete before the next begins. From the ξ reduction, the field's internal relaxation scale is $\ell_\xi = 1/\sqrt{\beta\sqrt{K}}$, which is microscopic (~10^{-5} m at galactic positions). The source variation scale L is astrophysical (~kpc). The condition $\ell_\xi \ll L$ is satisfied by tens of orders of magnitude. This IS the overdamped condition that licenses the first-order gradient equation. *Resolved through the ξ reduction.*

8.4 Term Mapping

Gradient term	CPTP origin	Covariant origin	Physical process
$-M®\cdot\rho_c$	$\Phi_{survive}$: Δ(ρ) erases off-diagonal	$\beta\sqrt{K}\cdot(\xi-1)$: curvature screens coupling	Phase exported to environment / curvature restores GR
R®	$\Phi_{recycle}$: Y fraction returned	$(\alpha/M^2_P)\cdot T$: matter sources coupling	Phase recycled / matter enhances coupling
$-A_\chi\tau$®	Overhead of recycling channel	Field self-energy back-reaction	Entropy cost of maintaining $\xi \neq 1$

8.5 Remaining Open Problem

The formal proof that the discrete CPTP iteration converges to the continuous gradient equation *independently* of the ξ reduction — that is, a purely bottom-up derivation from quantum information theory alone — remains open. This would require showing that the sequence of spatially indexed CPTP maps satisfies the conditions for a well-defined continuum limit (analogous to the Trotter product formula for time evolution).

This is a mathematical refinement, not a structural gap. The gradient equation is now derived from above (covariant field theory) and physically motivated from below (CPTP projection structure). A purely bottom-up derivation would complete the circle but would not change any physical prediction.

9. Born Rule from CPTP Structure [STANDARD QM + COMPANION PAPER]

9.1 Consistency Argument

The survival channel $\Phi_{survive} = \Delta$ (dephasing) maps:

$$|\psi\rangle\langle\psi| \mapsto \sum_i |\langle i|\psi\rangle|^2 |i\rangle\langle i| \tag{9.1}$$

The diagonal elements $p_i = |\langle i|\psi\rangle|^2$ are consistent with CPTP requirements:

- Non-negative (density matrix positivity) ✓
- Sum to 1 (trace preservation) ✓
- Continuous functions of ψ (CPTP maps are continuous) ✓
- Linear in ρ (Φ is linear) ✓

9.2 Connection to Uniqueness Proof

The full uniqueness proof — that $|\psi|^2$ is the ONLY quantity satisfying these requirements — is given in the companion Born Rule Uniqueness paper through the continuity equation argument (the source term $p(p-2)/(4m)|\psi|^{p-2}(\nabla S\cdot\nabla|\psi|^2)$ vanishes only for $p = 2$).

The CPTP framework provides a complementary consistency check: extracting $|\langle i|\psi\rangle|^p$ for $p \neq 2$ would produce a map that is either non-linear in ρ (violating CPTP linearity) or non-trace-preserving ($\Sigma_i |c_i|^p \neq 1$ for general states when $p \neq 2$).

Status: The Born rule is a consistency requirement of the CPTP structure. The full uniqueness proof is in the companion paper; here it is confirmed by the CPTP framework's constraints.

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10. Q ↔ M from the Stinespring Dilation [STRUCTURAL ARGUMENT]

10.1 The Core Observation

The Stinespring dilation provides a single unitary U_SE generating the projection. This unitary is generated by H_int, the system-environment interaction Hamiltonian (equation 4.2 in the Q ↔ M Equivalence paper):

$$H_{\text{int}} = \sum_i g_i \, |i\rangle\langle i|_S \otimes B_i^E \tag{10.1}$$

Tracing U_SE over different subsystems gives:

- **Trace over E → system-side effect:** decoherence dynamics, which in the Madelung picture manifest through the quantum potential Q resisting localization.
- **Trace over S → environment-side absorption:** phase information absorbed and partially fossilized, accumulating as M®.

10.2 What Is Established

Q and M arise from the same generating interaction H_int. They are structural counterparts — the system-side and environment-side signatures of the same physical process.

10.3 What Is Not Yet Established

The precise mathematical reduction showing how H_int generates the specific curvature term $\nabla^2\sqrt{\rho}/\sqrt{\rho}$ on the system side and the specific accumulation rate $dM/dt = (1-\Upsilon)\cdot\Gamma\cdot C$ on the environment side from the same operator is a structural argument, not a closed derivation.

The Q ↔ M Equivalence companion paper provides:

- A rigorous WKB consistency check ($Q \propto M^2$ in the slowly-varying regime)
- A virial verification (⟨Q⟩ = 13.6 eV for hydrogen)
- A constitutive closure mapping (approximate, not exact)
- An honest assessment of what remains open

Status: Conceptually powerful. Structurally consistent. Not yet a closed mathematical derivation.

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11. The Generative Triple

11.1 Definition

The framework's projection process is encoded in:

$$\mathcal{P} = \left(U_{SE}, \{|i\rangle\}, \Upsilon = \frac{2}{3} \right) \tag{11.1}$$

where:

- **U_SE** is the system-environment unitary (Stinespring dilation — standard QM)
- **{|i⟩}** is the pointer basis (einselected by H_int — standard QM)
- **ϒ = 2/3** is the recycling efficiency (thermodynamic hypothesis — ϒ Microphysics paper)

11.2 What P Generates

P is the **common generative mechanism** underlying the framework's predictions. The framework's results derive from P together with additional modeling steps (spatial iteration, cosmological integration, constitutive closures):

Framework element	Origin in P	Additional steps required
Born rule $P = \|\psi\|^2$	Φ_survive = Δ(ρ), standard QM	Uniqueness proof (companion paper)
Gradient equation	Spatial iteration of Φ_proj	Coarse-graining limit (§8, Supported by covariant reduction)
Suppression M®	(1−Y) × fossilization from Φ^c	Spatial accumulation model
Regeneration R®	Y × recycling from Φ^c	Collective feedback model
Bandwidth B_c(z)	1 − ∫ fossilization, exponent (1−Y) = 1/3	Cosmological integration
Q ↔ M equivalence	Complementary partial traces of U_SE	Constitutive closure (approximate)

11.3 What P Does Not Contain

P does not specify:

- **H_system** — the system Hamiltonian (input, not derived)
- **H_int structure** — the specific coupling form (determined by physics, selects pointer basis)
- **Spatial geometry** — the configuration over which iteration runs
- **Initial conditions** — ρ_c(0), M(0), etc. (cosmological boundary conditions)
- **Derivation of specific observational predictions** — these require additional modeling layers (Papers 1–17)

11.4 Honest Assessment

P is a **scaffold**, not a theory-of-everything generator. It provides:

1. A standard QM foundation (CPTP) ensuring compatibility with quantum mechanics ✓
2. A thermodynamically motivated branching hypothesis (Y = 2/3) ✓
3. A structural connection between quantum (Q) and classical (M) descriptions ✓
4. A framework within which all 17 papers' results can be embedded consistently ✓

It does NOT by itself derive cosmological evolution, dark matter profiles, or GR modifications. Those derivations exist in the individual papers and use P as their microphysical foundation together with additional modeling steps appropriate to each domain.

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12. Information-Theoretic Properties [STANDARD QM]

12.1 Channel Capacities

For Φ_proj with $\lambda = 1$ (complete projection):

Quantum capacity: Q(Φ_proj) = 0. No quantum information survives. Layer 1 has no quantum coherence.

Classical capacity: C(Φ_proj) = $\log_2$ d. Full classical information survives. Layer 1 retains all Born rule information.

Private capacity: P(Φ_proj) = 0. No private information — everything the system knows, the environment also knows (quantum Darwinism). Classical reality is objective.

12.2 Entropy Production

Per projection event:

$$\Delta S = S(\Delta(\rho)) - S(\rho) = H(\{p_i\}) - S(\rho) \geq 0 \tag{12.1}$$

with equality iff ρ is already diagonal. Projection always produces entropy — consistent with Second Law and irreversibility of Layer 0 → Layer 1 transition.

12.3 Monotonicity

Under iterated projection: $S(\rho\_c(r_2)) \geq S(\rho\_c(r_1))$ for $r_2 > r_1$. This encodes:

- Arrow of time (projection is irreversible)
- Suppression in gradient equation ($M \cdot \rho\_c \geq 0$)
- Bandwidth decrease with cosmic time ($B\_c(z_1) < B\_c(z_2)$ for $z_1 < z_2$)

All three are the data processing inequality applied to the projection channel.

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13. Comparison to Existing Frameworks

13.1 Zurek's Quantum Darwinism

The framework agrees completely with Zurek's einselection. Our $\Phi\_survive = \Delta$ IS Zurek's pointer-state selection, and our pointer basis $\{|i\rangle\}$ IS Zurek's pointer states.

What the framework adds: A specific model for the complementary channel's subsequent evolution (the Y split). Zurek describes how the environment monitors the system. The framework asks what happens to the environment afterward.

13.2 Decoherent Histories (Griffiths, Gell-Mann, Hartle)

The AHC (Admissible History Constraint) is closely related to consistent history selection. Both restrict which coarse-grained descriptions survive decoherence.

What the framework adds: The physical mechanism for history selection (pointer basis from H_int) and quantitative fossilization rate (1/3 per Landauer).

13.3 Objective Collapse Models (GRW, Penrose)

These modify the Schrödinger equation with stochastic collapse terms.

How the framework differs: No modification of Schrödinger. The CPTP map Φ_proj IS the "collapse" — standard decoherence, not a new dynamical mechanism. The Schrödinger equation remains exact at Layer 0.

13.4 Many-Worlds (Everett)

Many-worlds keeps unitary evolution and interprets all branches as real.

How the framework differs: The framework also keeps full unitarity (U_SE). But the fossilization channel at rate (1−Y) = 1/3 permanently removes interference capacity. Once phase information fossilizes into the medium (adding to M®), it cannot participate in future interference. This makes projection physically irreversible — not just "FAPP" (for all practical purposes) but structurally irreversible through information-theoretic capacity loss.

Important caveat: This irreversibility claim depends on the fossilization hypothesis (§6), which is the framework's additional physical content beyond standard QM. Under standard QM alone (without the Y hypothesis), the FAPP/true-irreversibility question remains open — exactly as in the standard many-worlds debate.

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14. Summary

14.1 What Is Established (Standard QM)

1. The projection process is a valid CPTP map (dephasing channel) ✓
2. Kraus decomposition exists and is correct ✓
3. Stinespring dilation provides the full system-environment unitary ✓
4. Complementary channel carries the system's lost phase information ✓
5. Born rule appears as the unique diagonal of the dephased density matrix ✓
6. Schrödinger equation is unmodified ✓
7. Entropy monotonically increases under projection ✓
8. Quantum Darwinism structure is correctly reproduced ✓

14.2 What Is Hypothesized (Framework Contribution)

9. The complementary channel splits into recycling (Y = 2/3) and fossilization (1/3) channels — justified by Landauer bound + detailed balance + temperature cancellation (Y Microphysics paper) ✓
10. Fossilization is physically irreversible — information capacity permanently lost ✓
11. Accumulated fossilization produces the suppression field M® ✓
12. Spatial iteration of the projection map yields the gradient equation (Supported by covariant reduction, empirically validated) ✓

14.3 What Is Open

13. Formal continuum limit of spatially indexed CPTP maps
14. Closed mathematical derivation of Q ↔ M correspondence
15. Derivation of specific H_int forms for different physical scales
16. Connection between pointer basis selection and coherence density formula

14.4 The Honest Claim

This paper provides:

A valid CPTP scaffold (standard QM) into which the framework's thermodynamic branching hypothesis can be embedded without violating quantum mechanics, together with a structural argument connecting quantum (Q) and classical (M) descriptions through the Stinespring dilation.

It is not yet a fully derived three-channel quantum instrument from first principles. But it is no longer metaphoric. It is structured in the language of quantum information theory, with explicit separation between standard results and physical hypotheses, and with all assumptions clearly labeled.

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